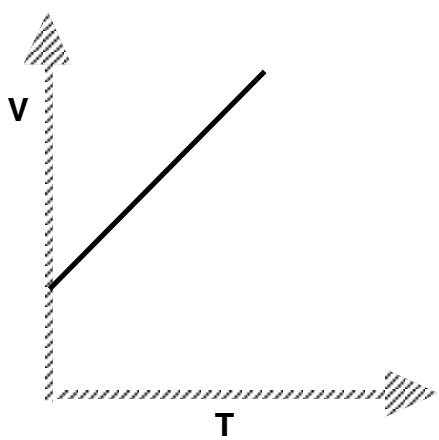
Unit I Reading – Graphical Methods

One of the most effective tools for the visual evaluation of data is a graph. The investigator is usually interested in a quantitative graph that shows the relationship between two variables in the form of a curve.

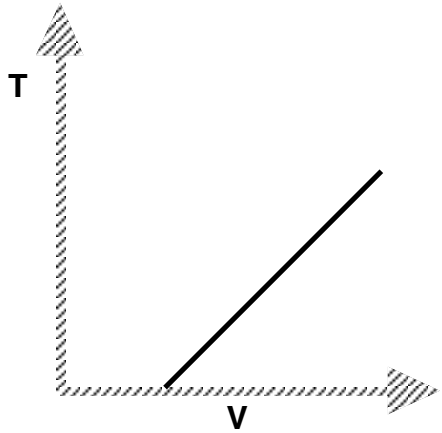
For the relationship y = f(x), **x** is the *independent variable* and **y** is the *dependent variable*. The rectangular coordinate system is convenient for graphing data, with the values of the dependent variable y being plotted along the *vertical axis* and the values of the independent variable x plotted along the *horizontal axis*.

Positive values of the dependent variable are traditionally plotted above the origin and positive values of the independent variables to the right of the origin. This convention is not always adhered to in physics, and thus the positive direction along the axes will be *indicated by the direction the arrow heads point.*

The choice of dependent and independent variables is determined by the experimental approach or the character of the data. Generally, the **independent variable** is the one over which the *experimenter has complete control*; the **dependent variable** is the one that *responds to changes* in the independent variable. An example of this choice might be as follows. In an experiment where a given amount of gas expands when heated at a constant pressure, the relationship between these variables, V and T, may be graphically represented as follows:



Correct



Incorrect

By established convention it is proper to plot V = f (T) rather than T = f (V), since the experimenter can directly control the temperature of the gas, but the volume can only be changed by changing the temperature.

Curve Fitting

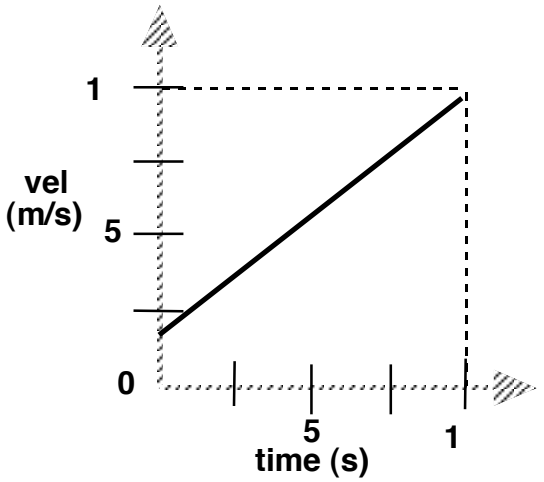
When checking a law or determining a functional relationship, there is good reason to believe that a uniform curve or straight line will result. The process of matching an equation to a curve is called **curve fitting**. The desired empirical formula, assuming good data, can usually be determined by inspection. There are other mathematical methods of curve fitting, however they are very complex and will not be considered here. Curve fitting by inspection requires an assumption that the curve represents a linear or simple power function.

If data plotted on rectangular coordinates yields a straight line, the function y = f(x) is said to be *linear* and the line on the graph could be represented algebraically by the slope-intercept form:

$$y = mx + b,$$

where **m** is the slope and **b** is y-intercept.

Consider the following graph of velocity vs. time:



The curve is a straight line, indicating that v = f(t) is a linear relationship. Therefore,

$$\mathbf{v} = m\mathbf{t} + b,$$

where slope $= m = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

From the graph,

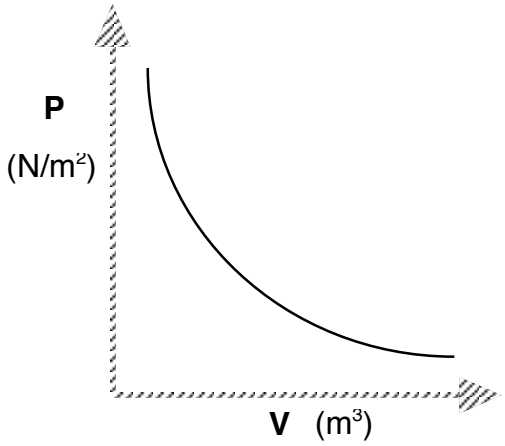
$$m = \frac{8.0 \text{ m/s}}{10.0 \text{ s}} = 0.80 \text{ m/s}^2 \ .$$

The curve intercepts the v-axis at v = 2.0 m/s. This indicates that the velocity was 2.0 m/s when the first measurement was taken; that is, when t = 0. Thus, $b = v_0 = 2.0$ m/s.

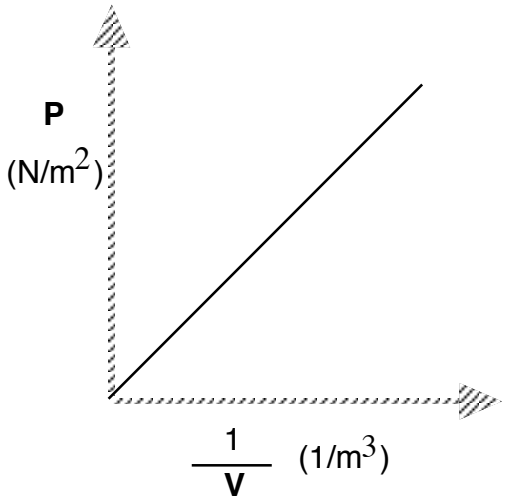
The general equation, $\mathbf{v} = m\mathbf{t} + b$, can then be rewritten as

$$\mathbf{v} = (0.80 \text{ m/s}^2\ )\mathbf{t} + 2.0 \text{ m/s}.$$

Consider the following graph of pressure vs. volume:



The curve appears to be a hyperbola (inverse function). Hyperbolic or inverse functions suggest a test plot be made of P vs $\frac{1}{V}$. The resulting graph is shown below:

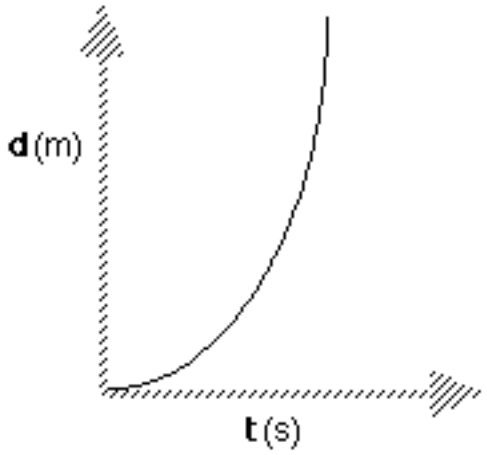


The equation for this straight line is:

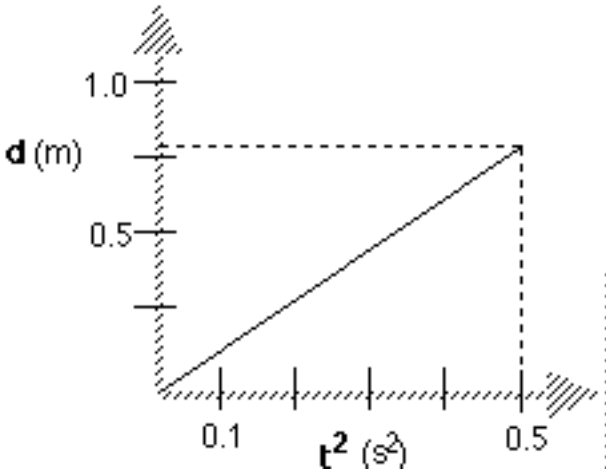
$$P = m\left(\frac{1}{V}\right) + b,$$

where b = 0. Therefore; $P = \frac{m}{V}$; when rearranged, this yields PV = constant, which is known as Boyle's law.

Consider the following graph of distance vs. time:



The curve appears to be a top-opening parabola. This function suggests that a test plot be made of d vs. t^2. The resulting graph is shown below:



Since the plot of d vs. t^2 is linear,

$$\mathbf{d} = m\mathbf{t^2} + b.$$

The slope, m, is calculated by

$$m = \frac{\Delta d}{\Delta t^2}$$

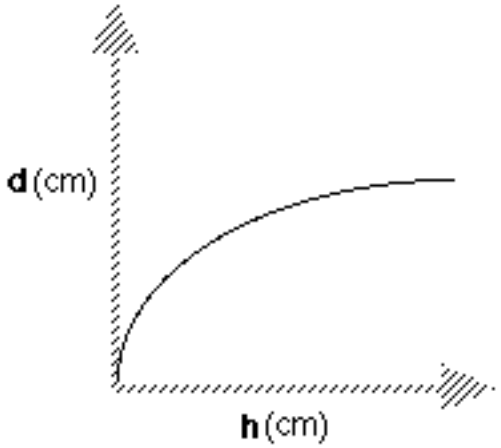
$$= \frac{.80m}{.50s^2}$$

$$= 1.6\ m/s^2$$

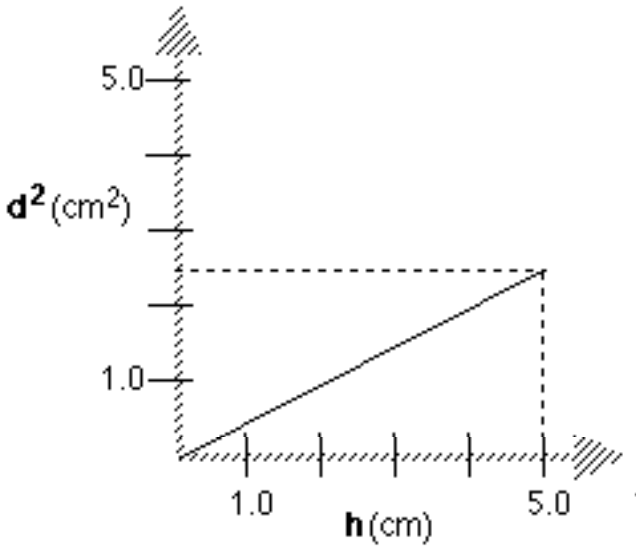
Since the curve passes through the origin, b = 0. The mathematical expression that describes the motion of the object is

$$\mathbf{d} = (1.6\ m/s^2)\mathbf{t^2} \ .$$

Consider the following graph of distance vs. height:



The curve appears to be a side-opening parabola. This function suggests that a test plot be made of d^2 vs. h. The resulting graph is shown on the following page.



Since the graph of d^2 vs. h is linear the expression is

$$\mathbf{d^2} = m\mathbf{h} + b.$$

The slope, m, is calculated by

$$m = \frac{\Delta d^2}{\Delta h}$$

$$= \frac{2.5 cm^2}{5.0 \text{ cm}}$$

$$= 0.50 \text{ cm}.$$

Since the curve passes through the origin, b = 0. The mathematical expression is then

$$\mathbf{d^2} = (0.50 \text{ cm})\mathbf{h}.$$